

# Models of Viscous Flow in the Earth’s Mantle With Constraints From Mineral Physics and Surface Observations

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## SUMMARY

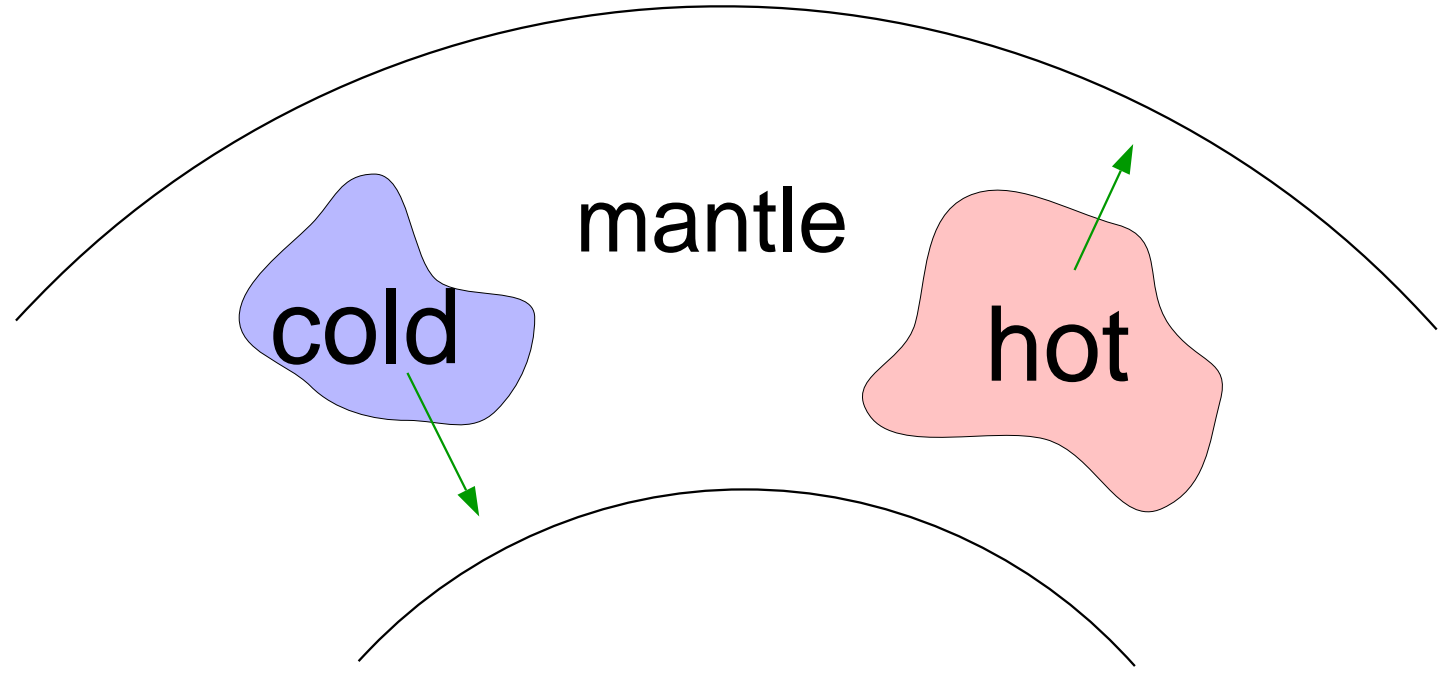
The rather poor knowledge of the scaling relation between seismic velocity anomalies and density anomalies, together with uncertainty in the mantle viscosity profile results in large uncertainties in models of mantle flow. To better constrain viscous flow models of mantle flow, we present and use mineral physics based models for both the scaling factor from seismic velocity to density anomalies and the radial viscosity structure. The scaling factor is derived from a mineral physics model for a uniform Pyrolite bulk composition mantle which satisfies the 1D seismic reference profiles of density and velocity across the entire depth extent of the mantle along a single adiabatic geotherm. Thus, there is no need to invoke chemical or thermal boundary layers anywhere within the mantle, and consequently, the scaling factor used here is purely a thermal conversion one. The calculated profile indicates that the scaling factor is between 0.2 and 0.3 in the bulk mantle (i.e. below the lithosphere, above D”). The model of viscosity structure and scaling factors is combined with ten different tomographic models to obtain mantle flow models that are subsequently used to make a number of predictions, which can be compared to observations: Here we focus on geoid, heat flux and lithospheric stresses.

The mineral physics models only constrain relative viscosity variations within each mantle layer (upper mantle, transition zone, lower mantle). By varying the “anchor viscosities” that define absolute viscosity values in each layer, we can optimize the fit to the geoid while additionally satisfying the “Haskell average” from postglacial rebound constraints and obtaining a reasonable advected heat flux profile. For a number of tomography models, we are able to obtain a geoid variance reduction of more than 70 per cent The resulting optimized viscosity profiles tend to increase from a minimum value of somewhat more than  $10^{20}$  Pas to about  $10^{22}$  Pas in the upper part of the lower mantle and about  $10^{23}$  Pas above D”. Depending on the tomography model, the viscosity minimum is either in the upper 400 km (asthenosphere) or the transition zone. In the first case, the mantle viscosity profile obtained is very similar to a profile previously proposed to explain hotspot tracks. Our predicted heat flux profiles tend to indicate that the mantle is heated largely from below, but because amplitudes vary between different recent tomographic models, they do not tightly constrain the fraction of internal heating of the mantle. Additionally, we compute the predicted lithospheric stresses for the optimized models and compare then with observed stress directions. Average angular misfits are frequently less than 30 degrees.

## Mantle flow models - general

Viscous flow in the Earth’s mantle is presumably the principal way how the Earth transports heat through the bulk mantle. Mantle flow can be computed, if we know the density field and viscosity structure. Input parameters can be constrained by comparing model predictions with “observations”:

- geoid and gravity
- advected heat flux
- plate motions
- dynamic topography
- lithospheric stress field
- hotspot tracks
- true polar wander
- ...



The model fit can be optimized within some parameter space, which should be kept small in order to avoid nonuniqueness.

## Specific features of work presented here

We keep the parameter space small by

- keeping the seismic velocity to density anomaly scaling factor profile fixed
- keeping the shape of viscosity profiles in the upper mantle, transition zone, lower mantle fixed, but allowing to vary “anchor viscosities”
- an additional “Haskell constraint” (from postglacial rebound)

Additionally, lithospheric viscosity is treated as free parameter. Thus, our most basic model has only three free parameters.

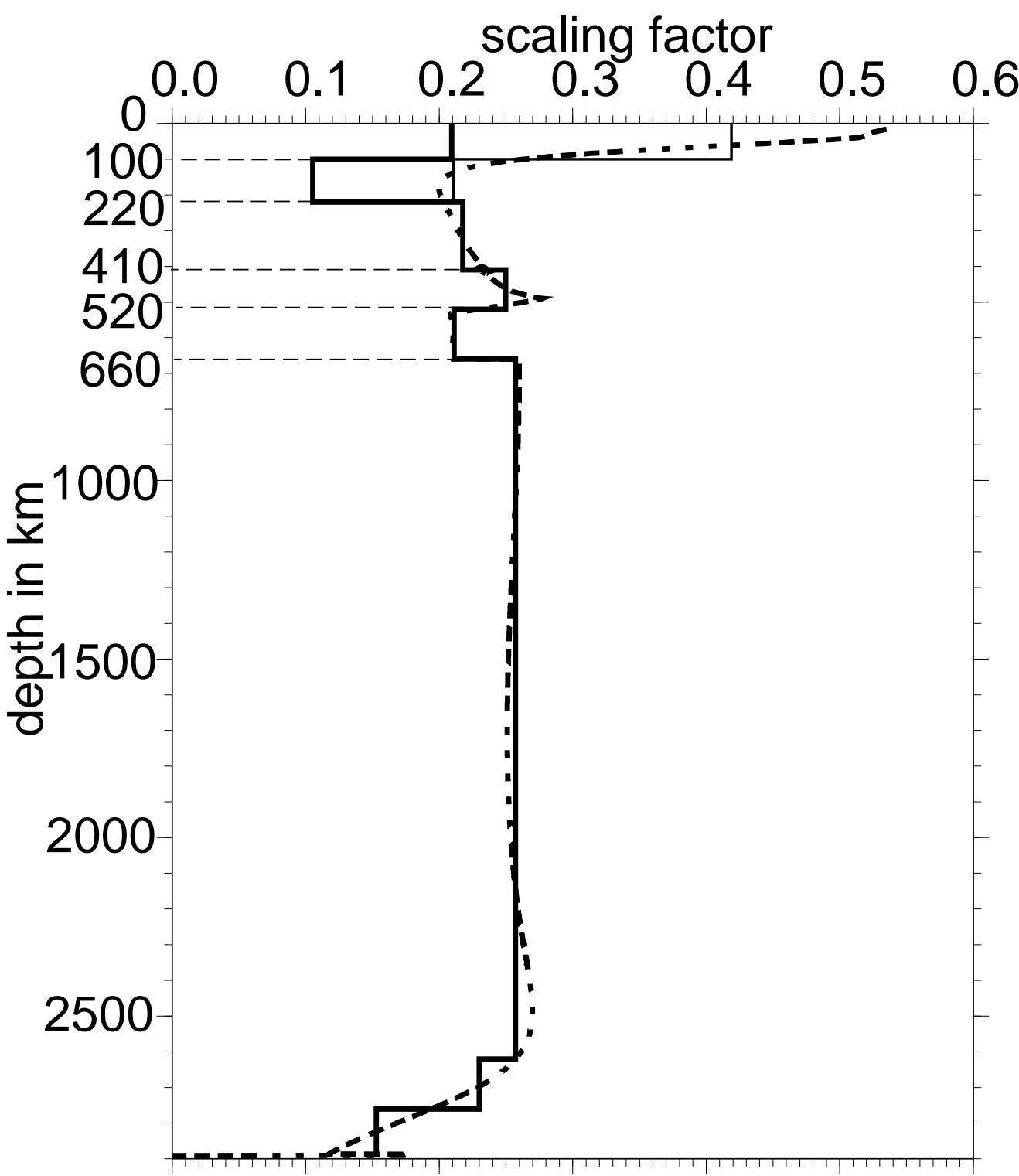
Geoid and heat flux profiles are used as observational constraints in the optimization of our model. A posteriori, we also compare predicted lithospheric stresses for optimized models with observations. A total of ten different s-wave tomography models are used to infer the density field.

## Seismic velocity to density anomaly scaling

The scaling factor profile presented here is based on the following assumptions:

- Pyrolite composition
- Density and seismic velocity variations are thermal in origin: Scaling factor  $(\delta\rho/\rho)/((\delta v_s/v_s) = (\alpha/\rho)/((dv_s/dT)_p/v_s)$
- A thermal expansivity profile  $\alpha(z) = \alpha(T(z), p(z))$  as described below
- A given temperature profile, which is adiabatic outside the boundary layers
- A  $(dv_s/dT)_p(z)$  - profile which includes the effects of anelasticity

In the top 220 km, scaling factors are arbitrarily reduced by a factor 2, because seismic anomalies are probably largely of chemical origin at shallow depth.



## Viscosity structure

An “effective viscosity”  $\eta = \sigma/\epsilon$  is derived from the stress-strain relationship

$$\dot{\epsilon} = C_1 \sigma^n \exp\left(-\frac{\Delta E + P\Delta V}{RT}\right),$$

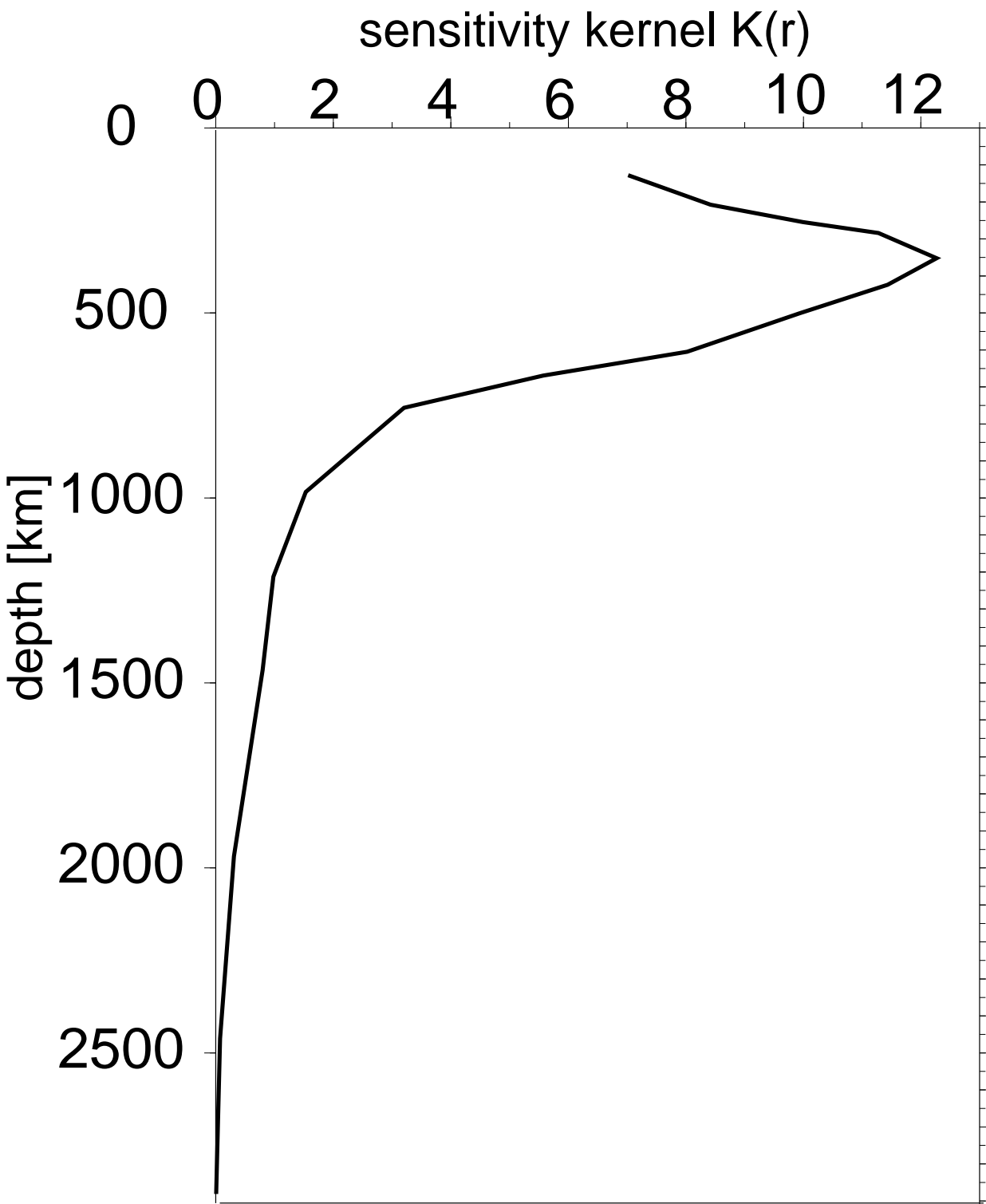
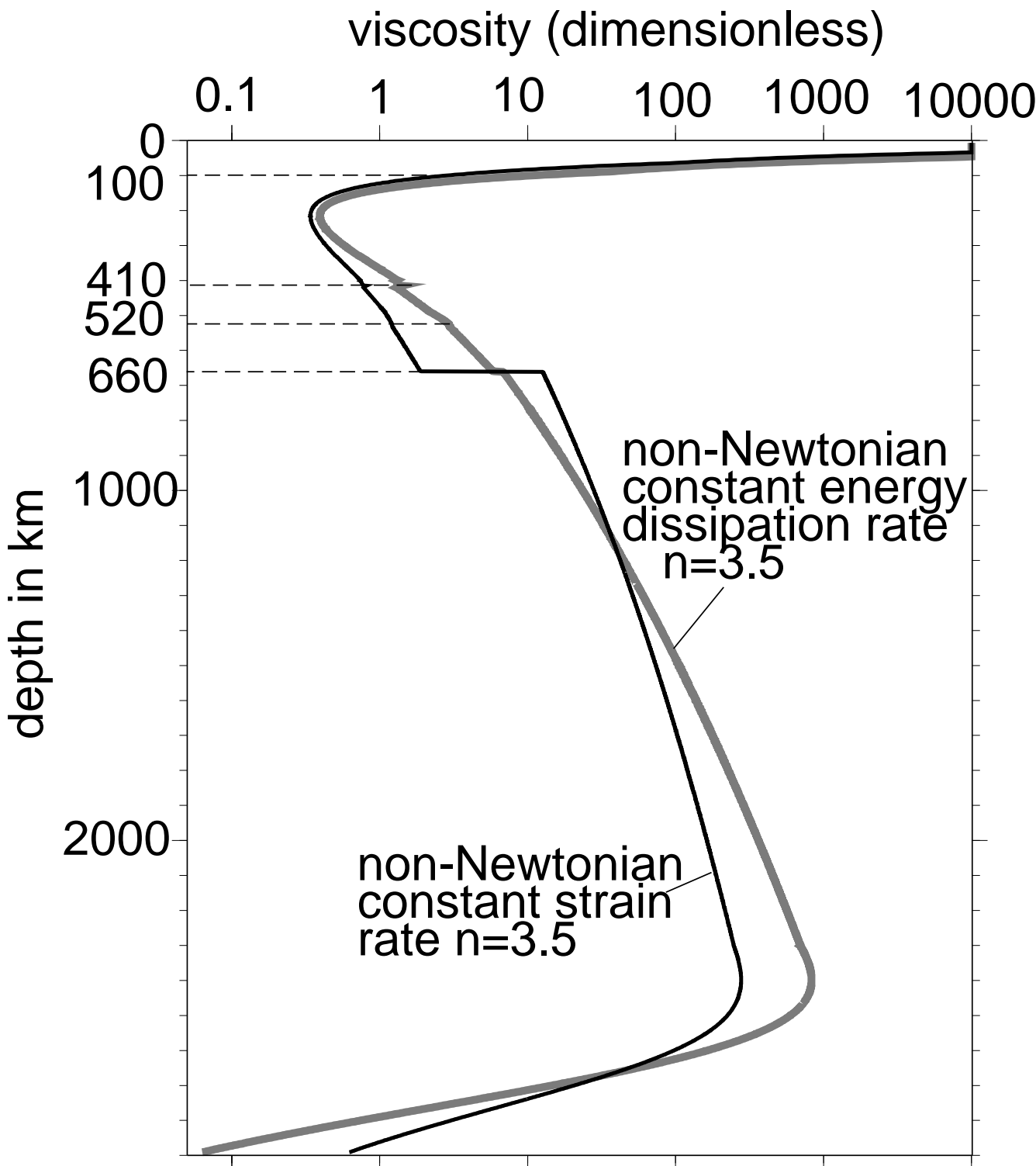
therefore for “constant strain rate”  $\sigma = \text{const.}$  an effective viscosity

$$\eta(z) = \eta(z_a) \exp\left(\frac{\Delta E + P\Delta V}{nRT(z)} - \frac{\Delta E + P\Delta V}{nRT(z_a)}\right)$$

is obtained, for “constant energy dissipation rate”  $\sigma\dot{\epsilon} = \text{const.}$  it is

$$\eta(z) = \eta(z_a) \exp\left(\frac{2(\Delta E + P\Delta V)}{(n+1)RT(z)} - \frac{2(\Delta E + P\Delta V)}{(n+1)RT(z_a)}\right),$$

A stress exponent  $n=3.5$  is used to derive  $\eta(z)$ , but for flow modelling lateral variations of  $\eta$  with  $\sigma$  and  $T$  will be neglected.



## Mantle flow and advected heat flux

Advected heat flux is computed as  $H(r) = \int_S \delta\rho \cdot v_r dA \cdot C_p/\alpha(r)$  with integration over the sphere of radius  $r$ .

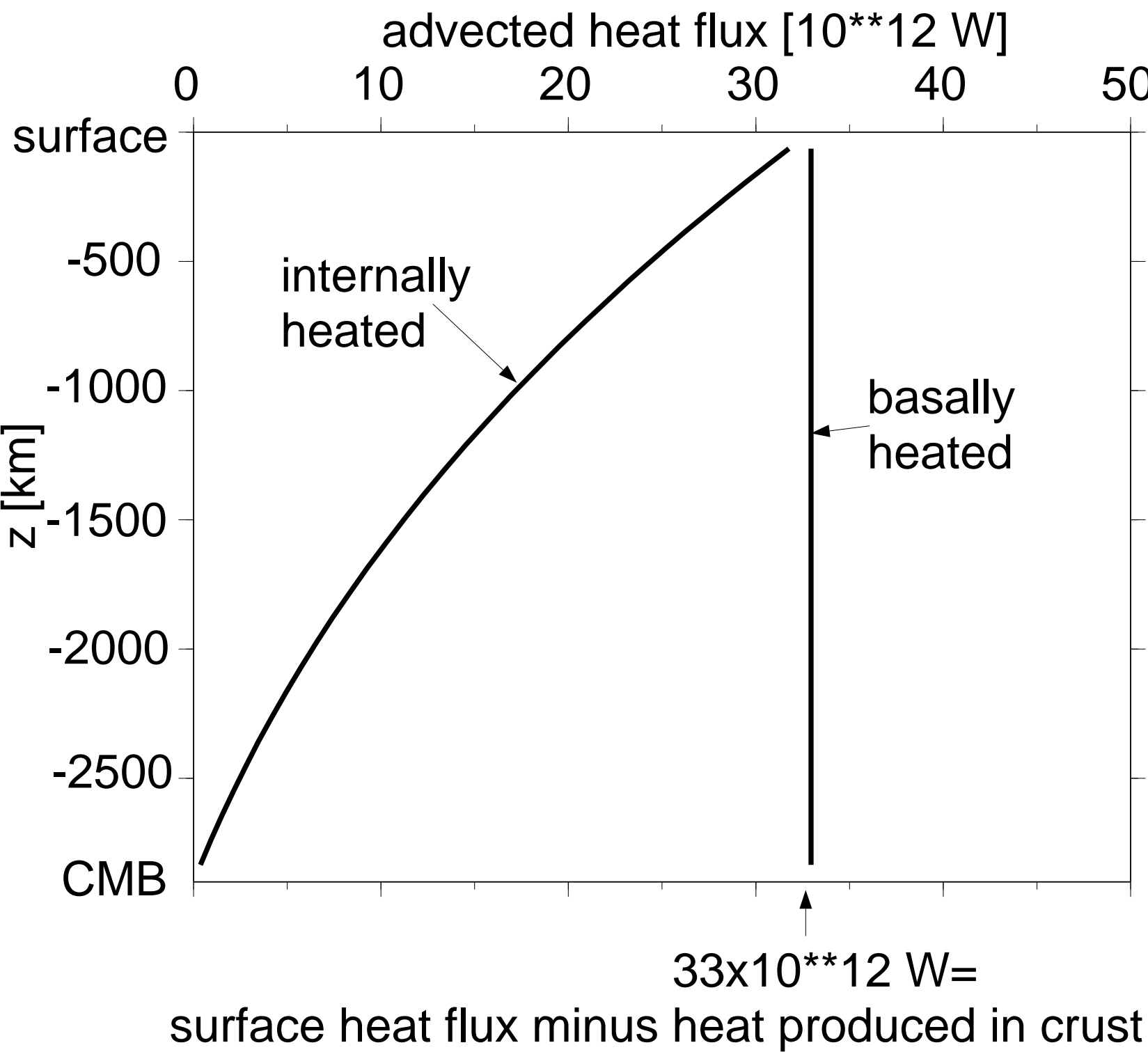
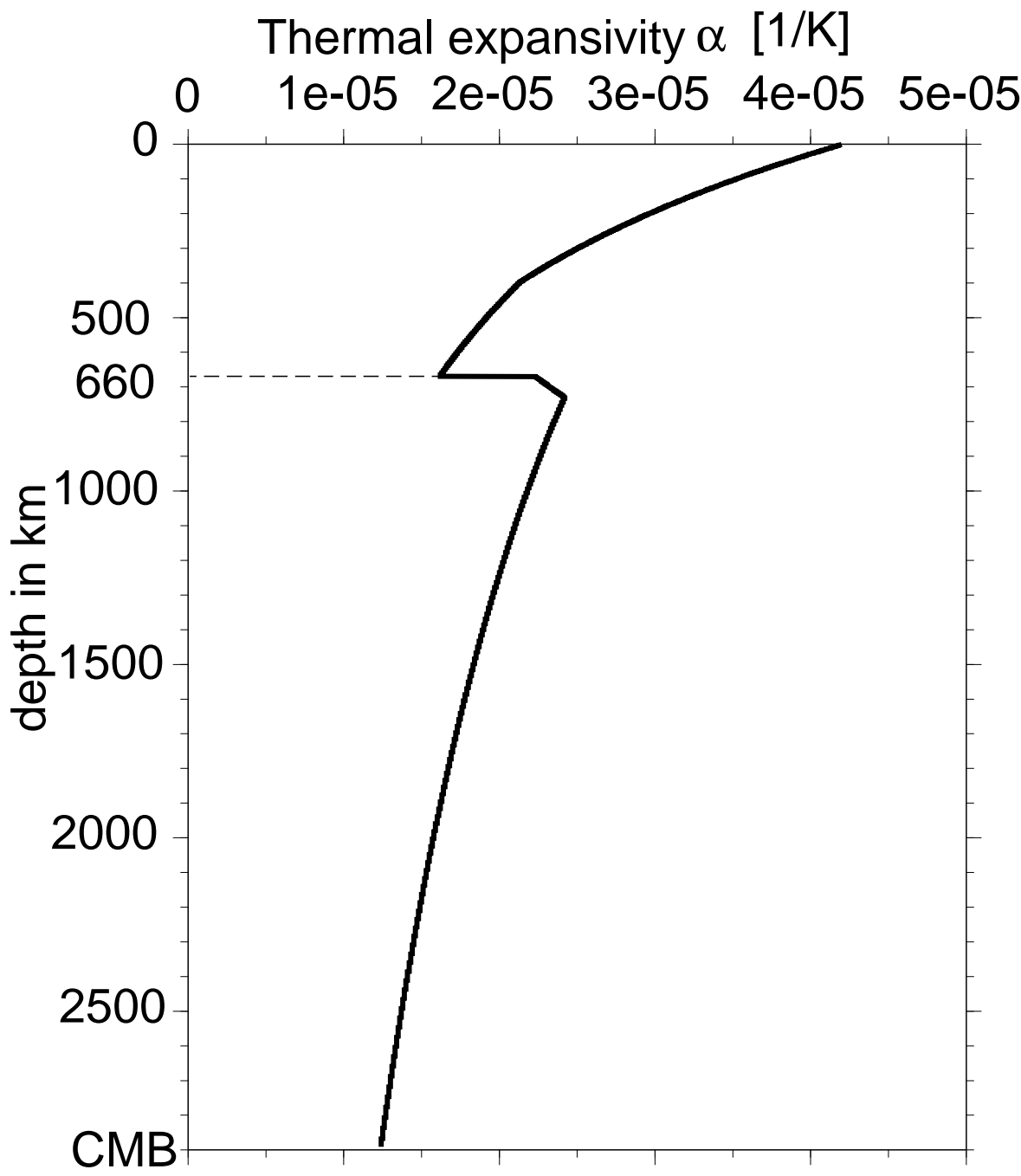
- The density anomaly  $\delta\rho$  is inferred from tomography models
- The radial velocity  $v_r$  follows from mantle flow calculation (with NUVEL plate motions as upper boundary condition)
- A constant heat capacity  $C_p = 1.25 \cdot 10^3$  J/(kg K) is assumed.
- The thermal expansivity profile  $\alpha(r)$  is computed for a depth-dependent phase assemblage for a pyrolite mantle and thermal expansivities of the individual phases

## “Haskell constraint”

The weighted volumetric average of viscosity is required to be equal to “Haskell value”  $10^{21}$  Pas (derived from postglacial rebound)

$$\int_{120\text{km}}^{\text{CMB}} \log(\eta(r)) \cdot r^2 \cdot K(r) dr / \int_{120\text{km}}^{\text{CMB}} r^2 \cdot K(r) dr = 21$$

The kernel shows the sensitivity of postglacial rebound to viscosity at various depths and is from Mitrovica (1996) for Angerman River



The predicted heat flux should approximately be between the theoretical steady-state curves for “basally heated” and “internally heated”

## Mantle flow and geoid anomalies

Both internal density heterogeneities and boundary deformations caused by induced flow contribute to geoid anomaly. The quality of the geoid prediction is frequently measured by variance reduction

$$VR = \frac{\text{Var}(\text{Predicted} - \text{Observed})}{\text{Var}(\text{Observed})}$$

The geoid is calculated for a free upper boundary.

